

# Analyzing Neutron Star in HESS J1731–347 from Thermal Emission and Cooling Theory

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Accepted . Received ; in original form

## ABSTRACT

The central compact object in the supernova remnant HESS J1731–347 appears to be the hottest observed isolated cooling neutron star. The cooling theory of neutron stars enables one to explain observations of this star by assuming the presence of strong proton superfluidity in the stellar core and the existence of the surface heat blanketing envelope which almost fully consists of carbon. The cooling model of this star is elaborated to take proper account of the neutrino emission due to neutron-neutron collisions which is not suppressed by proton superfluidity. Using the results of spectral fits of observed thermal spectra for the distance of 3.2 kpc and the cooling theory for the neutron star of age 27 kyr, new constraints on the stellar mass and radius are obtained which are more stringent than those derived from the spectral fits alone.

**Key words:** dense matter – equation of state – neutrinos – stars: neutron

## 1 INTRODUCTION

The neutron star XMMU J173203.3–34418 (hereafter XMMU J1732) belongs to the class of central compact objects (CCOs), relatively young cooling neutron stars in supernova remnants. CCOs possess relatively low dipole-like large-scale surface magnetic fields ( $\lesssim 10^{10} - 10^{12}$  G) and show thermal emission in soft X-rays. XMMU J1732 was discovered in 2007 with *XMM-Newton* (Tian et al. 2010) near the center of the HESS J1731–347 (= G353.6–0.7) supernova remnant. It was observed also with *Suzaku* and *Chandra* (e.g. Halpern & Gotthelf 2010). Spectral fits of the X-ray emission from XMMU J1732 with traditional black-body and hydrogen atmosphere models yielded radius of the emitting region much lower than typical radii of neutron stars for the plausible distance to the source of 3–5 kpc. These results might have been interpreted as the radiation from a hot spot on the neutron star surface. However, no pulsations have been detected from XMMU J1732 so far which would imply a very special geometry in which either the spot is aligned with the spin axis of the star or the spin axis is directed along the line of sight.

This uncomfortable explanation was questioned by Klochkov et al. (2013). The authors fitted the spectra with the carbon atmosphere model and showed that such fits lead to the radius of the emitting region comparable to a typical radius of

neutron stars. If so, the observed radiation can be interpreted as the thermal radiation emergent from the entire neutron star surface. We adopt this explanation here. We note, that XMMU J1732 is the second CCO where the carbon atmosphere is used to explain the observed emission properties, after the neutron star in the Cassiopeia A supernova remnant (Ho & Heinke 2009).

The next important step was done by Klochkov et al. (2015), hereafter Paper I, who analyzed longer observations of XMMU J1732 with *XMM-Newton* and fitted the entire dataset with the carbon atmosphere model computed by Suleimanov et al. (2014). They analyzed different assumptions on the distance  $d$  to the star and concluded that  $d = 3.2$  kpc seems most realistic (although other possibilities are not excluded). Assuming  $d = 3.2$  kpc they obtained sufficiently narrow confident regions for the neutron star mass and radius. As for the neutron star age  $t$ , they took  $t = 27$  kyr from Tian et al. (2008) and added (rather arbitrarily) an uncertainty range of 10–40 kyr as the age might in fact be lower. We adopt these values here and apply the elaborated cooling model to analyze the data.

It is well known that the cooling theory allows one to study the properties of superdense matter in neutron star cores and constrain the parameters of neutron stars (e.g., Yakovlev & Pethick 2004; Page et al. 2009). Here, we continue the theoretical interpretation of XMMU J1732 with the cooling theory started in Paper I but using a more elaborated cooling model.

Among cooling isolated middle-aged neutron stars whose thermal surface radiation has been detected, XMMU J1732 is a

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**Table 1.** Four models of XMMU J1732 used in fig. 7 of Paper I and in Fig. 1. MU, nn, np, and pp indicate the neutrino cooling due to modified Urca (MU) process, as well as due to three weaker reactions of nucleon-nucleon collisions. The sixth column indicates the composition of the heat blanketing envelope. The last column gives the theoretical surface temperature  $T_s^\infty$  at  $t = 27$  kyr. Only the last (SFac) model is consistent with observations. See text for details.

Model	MU	nn	np	pp	Heat blanket	$T_s^\infty$ [MK]
MU	on	on	on	on	iron	0.96
MUac	on	on	on	on	carbon	1.23
SF	off	on	off	off	iron	1.34
SFac	off	on	off	off	carbon	1.77

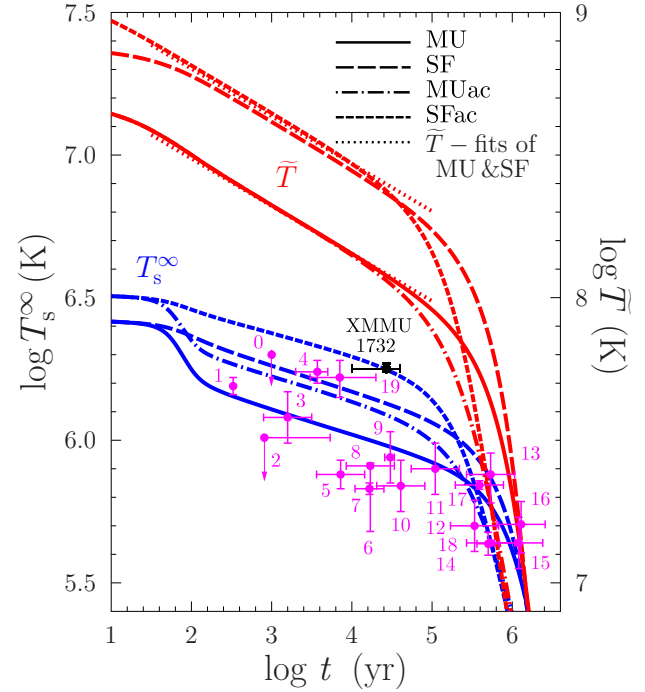
special case. At  $d = 3.2$  kpc the effective surface temperature, as measured by a distant observer, is  $T_s^\infty \sim 2$  MK (see Paper I and references therein). If  $t = 27$  kyr, XMMU J1732 is the hottest **cooling** neutron star with the measured surface temperature. Of course, we do not mean magnetars which can be hotter because of their magnetic activity.

This special status of XMMU J1732 is illustrated in Fig. 1 (after fig. 7 of Paper I) which shows the measured temperatures  $T_s^\infty$  (left vertical scale) and ages  $t$  of cooling neutron stars. The data and neutron star labels are the same as in Paper I, (0) the Crab pulsar; (1) the neutron star in Cas A; (2) the neutron star in 3C 58; (3) PSR J1119–6127; (4) RX J0822–43; (5) PSR J1357–6429; (6) RX J0007.0+7303; (7) the Vela pulsar; (8) PSR B1706–44; (9) PSR J0538+2817; (10) PSR B2334+61; (11) PSR 0656+14; (12) Geminga; (13) PSR B1055–52; (14) RX J1856–3754; (15) PSR J2043+2740; (16) RX J0720.4–3125; (17) RX J1741–2054; (18) PSR J0357+3205; (19) 1E 1207–52.

The data are compared with four theoretical cooling curves which correspond to the four cooling models of XMMU J1732 indicated in Table 1 and detailed below. These cooling curves have been calculated in Paper I for a neutron star with the gravitational mass  $M = 1.5 M_\odot$  and circumferential radius  $R = 12.04$  km (with a typical APR I equation of state – EOS – in the stellar core, Gusakov et al. 2005). The four lower curves show the evolution of the surface temperature  $T_s^\infty(t)$ , while the four upper curves show the evolution of the redshifted temperature  $\bar{T}(t)$  in the stellar centre. The surface temperature inferred from observations of XMMU J1732 is  $T_s^\infty = 1.78^{+0.04}_{-0.02}$  MK (Paper I); it can be compared with the theoretical values ( $t = 27$  kyr) presented in Table 1 for the four models.

The hot XMMU J1732 can be explained by the standard cooling theory although with great difficulty. It is thought to be an isolated neutron star born hot in a supernova explosion. It gradually cools down, and the cooling depends on properties of matter and neutron star parameters. The cooling theory (e.g. Yakovlev & Pethick 2004) states that XMMU J1732 is at the neutrino cooling stage with isothermal interior; its internal thermal relaxation should be over. Such a star should mainly cool via the neutrino emission from its super-dense core (e.g. Yakovlev et al. 2001). As in Paper I, we assume that the core consists of nucleons, electrons and muons, and the nucleons can be in superfluid state. It is well known (e.g., Lombardo & Schulze 2001) that protons in the core are usually paired in singlet state while neutrons can be paired in triplet state; singlet-state pairing of neutrons can occur in the neutron star crust.

As the internal layers of XMMU J1732 have to be isothermal, a substantial temperature gradient is kept only in a thin (no thicker than a few tens m) heat blanketing envelope. A large-scale surface



**Figure 1.** The effective surface temperatures  $T_s^\infty$  or upper limits (left vertical scale) for a number of isolated neutron stars including XMMU J1732 versus their ages (data points; see the text). The data are compared with the four theoretical cooling curves for a  $1.5 M_\odot$  neutron star (Paper I). Curves MU and SF refer to neutron stars without superfluidity and with strong proton superfluidity, respectively, which have iron heat blanketing envelopes. Curves MUac and SFac refer to similar neutron stars but with carbon envelopes. The four upper curves show evolution of the internal temperature  $\bar{T}$  (right vertical scale) for the same cooling scenarios. Two dotted curves are analytic approximations of  $\bar{T}$  described in Section 2.

magnetic field  $B \lesssim 10^{12}$  G which can exist in XMMU J1732 (Paper I) cannot greatly affect the cooling of this star. While analyzing the cooling it can be neglected.

Because XMMU J1732 is very hot, its cooling must be extremely slow. Theoretically, such a cooling can be explained (Paper I) by (i) a strong suppression of neutrino emission from the stellar core and (ii) assuming a massive carbon heat blanketing envelope. These two cooling regulators are most important here.

(i) No enhanced neutrino cooling mechanism such as direct Urca process (Lattimer et al. 1991) or neutrino emission due to triplet-state Cooper pairing of neutrons (Flowers, Ruderman & Sutherland 1976; Leinson & Pérez 2006; also see Page et al. 2009 and references therein) can operate in the XMMU J1732 core. The enhanced emission would make the star colder than it is. In particular, the core cannot contain any wide layer of superfluid neutrons.

Moreover, even if the standard neutrino emission due to the modified Urca (MU) process operated in the stellar core, the star would have been insufficiently hot. This is shown by the solid cooling curve MU in Fig. 1 and demonstrated as model MU in Table 1. The star is supposed to have the standard heat blanketing envelope made of iron (with a thin carbon atmosphere on top). According to Fig. 1 and Table 1, the MU model cannot explain the observations of XMMU J1732. The model implies that the star is non-superfluid; in addition to the MU processes, weaker processes of neutrino emission in neutron-neutron (nn), neutron-proton (np) and

proton-proton (pp) collisions (called neutrino-pair bremsstrahlung in nucleon-nucleon collisions) also operate in the core but do not affect the cooling.

The neutrino emission of the star can be further reduced by strong proton superfluidity in the stellar core. It greatly suppresses the neutrino reactions involving protons which are the MU-process and the weaker neutrino processes of pp and np collisions. However, the neutrino emission of non-superfluid neutrons in nn collisions survives; it is 30–100 times weaker than the MU process. The cooling scenario SF is for the star with strong proton superfluidity and iron heat blanket. The star becomes noticeably hotter than the MU star, but still insufficiently hot to explain the data.

(ii) The second powerful regulator of the XMMU J1732 surface temperature is the amount of carbon (generally, of accreted matter containing light elements – Potekhin, Chabrier & Yakovlev 1997) in the heat blanketing envelope. The thermal conductivity of the carbon envelope is higher than that of the iron one; at the same internal temperature the surface temperature becomes higher. Model MUac is for a non-superfluid star with nearly maximum possible mass of carbon  $\Delta M_C$  in the envelope ( $\Delta M_C \sim 10^{-8}M$ ). The surface temperature of a non-superfluid star becomes noticeably higher than that for the MU model, but it is still not high enough. Finally, model SFac is for the superfluid star with the maximum mass of carbon in the envelope. The double effect – of proton superfluidity and carbon envelope – makes the surface temperature exceptionally high (Fig. 1, Table 1), in agreement with observations.

Notice that the internal temperature of XMMU J1732 is almost independent of the composition of the heat blanket; in particular, the  $\tilde{T}(t)$  curves MU and MUac in Fig. 1 almost coincide at  $t \sim 100 \text{ yr} - 100 \text{ kyr}$ , as well as the curves SF and SFac. This is because XMMU J1732 cools via neutrinos from inside. Accordingly, the presence of carbon in the heat blanket just increases  $T_s^\infty$  making the surface hotter without any back reaction on the neutron star interiors.

It is worth to mention that the cooling scenarios SF and SFac can also be realized if direct Urca process is formally allowed in XMMU J1732 for a given EOS but is exponentially suppressed by very strong proton superfluidity (similar situations are described, e.g., by Yakovlev & Pethick 2004).

We analyse the XMMU J1732 cooling below. In two appendices we discuss the parameters of heat blanketing envelopes and the neutrino emission of the neutron star crust.

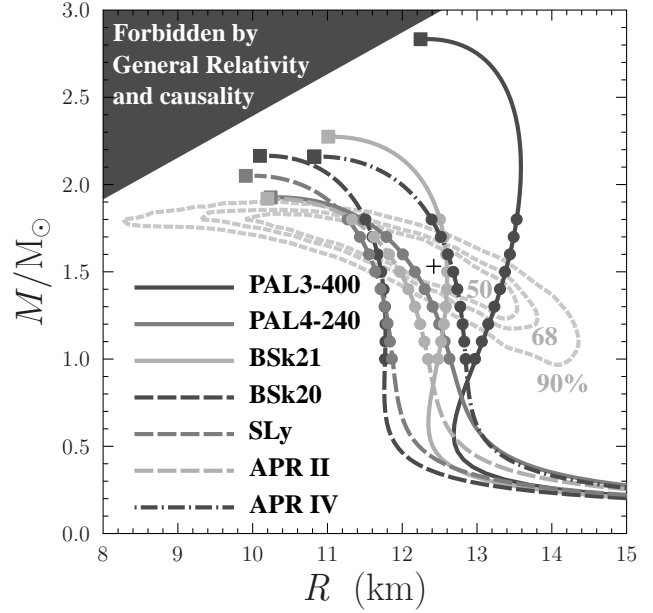
## 2 NEUTRINO COOLING FUNCTION OF NEUTRON STARS

Let us outline calculation of cooling curves,  $T_s^\infty(t)$ , for analyzing the XMMU J1732 cooling. The surface temperature  $T_s^\infty$  is directly related to the internal temperature (Yakovlev et al. 2011).

The decrease of the internal temperature of a neutron star at the neutrino cooling stage after reaching the internal thermal relaxation is described by the well known equation (e.g., Yakovlev et al. 2011)

$$\frac{d\tilde{T}}{dt} = -\ell(\tilde{T}), \quad \ell(\tilde{T}) = \frac{L_\nu(\tilde{T})}{C(\tilde{T})}. \quad (1)$$

Here,  $\tilde{T} = T\sqrt{g_{00}}$  is the redshifted temperature of the isothermal region within the star (it is this temperature which is constant in isothermal regions in the frame of General Relativity);  $T$  is the local (non-redshifted) temperature,  $g_{00}$  is the metric component in a



**Figure 2.**  $M - R$  relations for neutron star models with the selected EOSs. Filled squares refer to maximum mass models. Filled dots indicate neutron star models used for calculating  $q_{\text{MU}}$  and  $q_{\text{SF}}$  and for deriving the fit expressions (6) and (8). Dashed contours are confidence regions for XMMU J1732 obtained in Paper I at 50, 68 and 90 per cent significance levels by fitting the observed spectra with the carbon atmosphere models. The cross is the best fit. The shaded upper left corner is prohibited by General Relativity and causality. See text for details.

given point of the star;  $L_\nu$  and  $C$  are, respectively, the redshifted integral neutrino luminosity and heat capacity of the star determined mainly by the super-dense core, and  $\ell(\tilde{T})$  is the neutrino cooling function (e.g., Yakovlev et al. 2011). This function regulates the internal cooling at the neutrino cooling stage.

We have calculated  $\ell(\tilde{T})$  for two cases. In the first case, the star is assumed to be non-superfluid and cools mainly via the MU process; such stars can be called standard neutrino candles. In the second case, the star possesses very strong proton superfluidity with the critical temperature for this superfluidity  $T_{\text{cp}} \gtrsim 5 \times 10^9 \text{ K}$  everywhere in the core. Such superfluidity completely suppresses the MU, np and pp processes of neutrino cooling so that the star cools via the neutrino emission in nn collisions. In addition, strong proton superfluidity fully suppresses the proton heat capacity. In both cases  $L_\nu \propto \tilde{T}^8$  and  $C \propto \tilde{T}$ . Then

$$\ell(\tilde{T}) = q \tilde{T}_9^7. \quad (2)$$

The factor  $q$  (to be expressed in  $\text{K s}^{-1}$ ) depends on  $M$ ,  $R$  and EOS in the stellar core (see below);  $\tilde{T}_9 = \tilde{T}/(10^9 \text{ K})$ .

The cooling equation (1) with the neutrino cooling function (2) after reaching the state of internal thermal relaxation of the star gives the well known analytic solution (e.g. Yakovlev et al. 2011)

$$\tilde{T}_9 = (6qt/T_*)^{-1/6}, \quad (3)$$

where  $t$  has to be expressed in seconds; we insert the normalization temperature  $T_* = 10^9 \text{ K}$  to ensure proper dimensions of  $q$  and  $t$ .

The factor  $q = q(M, R)$  depends on proton superfluidity in a neutron star core. We will see that the factors for non-superfluid stars ( $q = q_{\text{MU}}$ ) and for the stars with very strong proton superfluidity ( $q = q_{\text{SF}}$ ) are related as  $q_{\text{SF}} \sim (0.01 - 0.02) q_{\text{MU}}$ . In reality,

proton superfluidity only partly suppresses the neutrino emission in reactions involving protons.

Following Yakovlev et al. (2011) we write

$$q = q_{\text{MU}} f_\ell, \quad f_\ell \equiv \ell(\tilde{T})/\ell_{\text{MU}}(\tilde{T}), \quad (4)$$

where  $f_\ell$  is the neutrino cooling function of the star expressed in terms of  $\ell_{\text{MU}}(\tilde{T})$  for a standard neutrino candle (Paper I); it is nearly temperature independent and small (for XMMU J1732 in our model). Evidently, we have  $q \geq q_{\text{SF}}$ . It is instructive to rewrite (4) in the form

$$q = f_{\text{tp}} q_{\text{MU}} + q_{\text{SF}}. \quad (5)$$

In this case,  $f_{\text{tp}} = f_\ell - f_{\text{tn}}$ , with  $f_{\text{tn}} \equiv q_{\text{SF}}/q_{\text{MU}}$  being the minimum neutrino cooling function of XMMU J1732 expressed in standard neutrino candles. Thus defined,  $f_{\text{tp}}$  describes an incomplete suppression of the standard neutrino candle emission by proton superfluidity. Such a suppression is determined by the density profile of the critical temperature for proton superfluidity  $T_{\text{cp}}(\rho)$  in the stellar core. Both quantities,  $T_{\text{cp}}(\rho)$  and  $f_{\text{tp}}$ , are a priori unknown.

Equations (4) and (5) give two equivalent methods to describe the effect of strong proton superfluidity on neutron star cooling:

(i) One can use (4) and describe the effect of proton superfluidity by the factor  $f_\ell$  which is restricted ( $f_\ell \geq f_{\text{tn}}$ ) by the neutrino cooling function due to non-superfluid neutrons. This approach was used in Paper I although  $f_{\text{tn}}$  was not accurately determined there. In principle,  $f_\ell$  might also be restricted by the neutrino emission from the neutron star crust but the latter restriction is insignificant (Appendix B).

(ii) Alternatively, one can employ (5) and characterize the effect of proton superfluidity by the factor  $f_{\text{tp}}$  defined in such a way that  $f_{\text{tp}} \rightarrow 0$  in the limit of very strong superfluidity. We adopt this approach here.

Notice that this refinement of the cooling theory is required only for very slowly cooling neutron stars ( $f_\ell \ll 1$ ). For other stars,  $f_\ell$  is much higher than  $f_{\text{tn}}$  so that  $f_{\text{tn}}$  can be disregarded.

In any case, (i) or (ii), we need  $q(M, R)$  in the two limits, for fully non-superfluid stars ( $q = q_{\text{MU}}$ ) and for the stars with very strong proton superfluidity in the core ( $q = q_{\text{SF}}$ ). We have calculated  $q_{\text{MU}}$  and  $q_{\text{SF}}$  for seven EOSs of superdense matter in neutron star cores. The SLy, PAL4-240 and PAL3-400 EOSs are described by Yakovlev et al. (2011), APR II is described by Gusakov et al. (2005); BSk20 and BSk21 by Potekhin et al. (2013), and APR IV EOS by Kaminker et al. (2014) (who called it the HHJ EOS). The  $M(R)$  relations for neutron star models with these EOSs are plotted in Fig. 2. The majority of these EOSs cover the range of  $M$  and  $R$  values which is usually treated as realistic (Haensel, Potekhin & Yakovlev 2007). However, some of the selected EOSs (e.g. PAL3-400) are purely phenomenological and are thought to be less realistic. They are included to enlarge the range of neutron star radii  $R$  involved in our analysis. Squares in Fig. 2 refer to most massive stable neutron star models. One can see that the chosen EOSs are reasonably consistent with recent observations of two massive ( $M \approx 2 M_\odot$ ) neutron stars (Demorest et al. 2010; Antoniadis et al. 2013). The calculations of  $q = q_{\text{MU}}$  and  $q = q_{\text{SF}}$  have been performed for a range of masses  $M=1.0, 1.1, 1.2, \dots, 1.8 M_\odot$ ; 63 calculated models are shown by filled dots. The majority of the selected EOSs open direct Urca process in sufficiently massive stars. While computing  $q(M, R)$  we have artificially switched off fast neutrino cooling due to direct Urca process. This allows us to include the case of switched on direct Urca process when it is exponentially suppressed by strong proton superfluidity

(Section 1). In this case  $q_{\text{MU}}$  determines a formal standard-candle neutrino emission level, our convenient unit for measuring the real level in hot stars like XMMU J1732. Notice that the values  $q(M, R)$  are calculated using the same effective masses of nucleons and matrix elements of neutrino reactions in neutron star cores which were used by Yakovlev et al. (2011).

We have approximated numerical values of  $q_{\text{MU}}$  by the expression

$$q_{\text{MU}}(M, R) = 4.59 \text{ K s}^{-1} \frac{\gamma^{10}}{1 + 0.3\gamma} \exp\left(0.16 \frac{\beta}{x}\right), \quad (6)$$

where

$$\gamma = \frac{1}{\sqrt{1-x}}, \quad x = \frac{r_g}{R}, \quad \beta = \frac{3M}{4\pi R^3 \rho_0}, \quad (7)$$

$r_g = 2GM/c^2 = 2.95 M/M_\odot \text{ km}$  is the Schwarzschild radius and  $\rho_0 = 2.8 \times 10^{14} \text{ g cm}^{-3}$  is the density of saturated nuclear matter. The root mean square (rms) relative fit error is 0.12, and the maximum relative error is 0.22 for the  $M = 1.8 M_\odot$  neutron star with the SLy EOS.

For the case of extremely strong proton superfluidity we have derived a similar approximation,

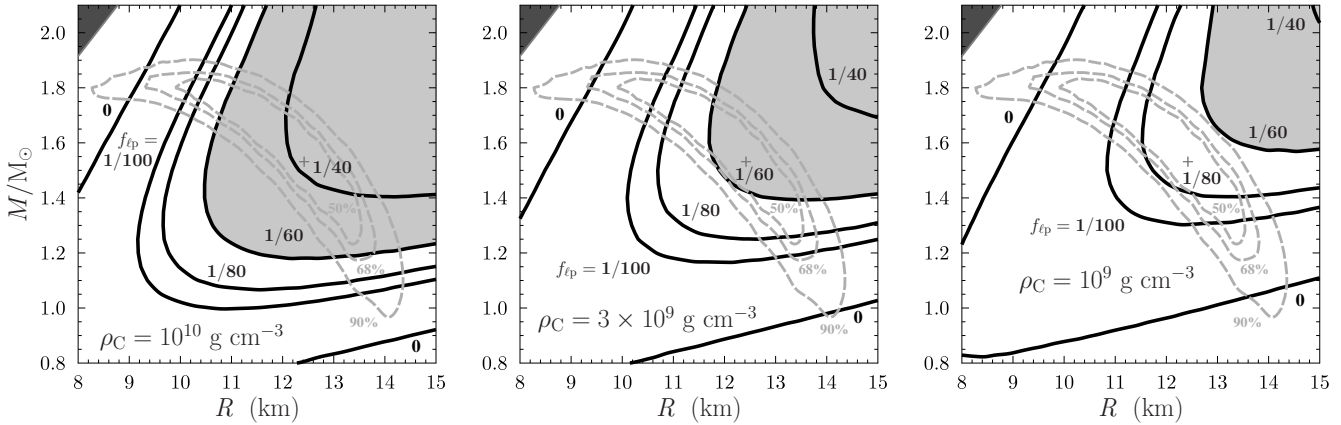
$$q_{\text{SF}} = 0.174 \text{ K s}^{-1} \frac{\gamma^5 \exp[0.624(\gamma-1)(1+1.03\beta)]}{1+0.0146\beta}. \quad (8)$$

Here the rms relative fit error is 0.016 and the maximum relative error 0.035 takes place for the  $1.8 M_\odot$  star with the BSk21 EOS. Notice that  $q_{\text{MU}}$  was approximated earlier by Yakovlev et al. (2011). Their fit is somewhat less accurate but covers larger range of masses,  $1 \leq M/M_\odot \leq 2.4$ . In our notations, their rms relative fit error of  $q_{\text{MU}}$  is 0.21, and the maximum error is 0.42. We have checked that for  $10 \leq R \leq 14 \text{ km}$  and  $1 \leq M/M_\odot \leq 1.8$  the rms relative deviation of  $q_{\text{MU}}$  values provided by the previous and new fits is about 0.24, and the maximum deviation  $\sim 0.56$  is reached at  $M = 1.0 M_\odot$  and  $R = 14 \text{ km}$ . As follows from a discussion below, such an agreement is quite satisfactory. We have constructed a new fit to make the fitting of  $q_{\text{MU}}$  and  $q_{\text{SF}}$  more uniform and to reduce the fit error. We stress that both approximations, (6) and (8), are valid for all selected EOSs. In this sense they are universal. The fit errors reflect deviations from universality.

The applicability of our fits (6) and (8) is greatly affected by a strong temperature dependence of the neutrino cooling function (2). If one uses (3) to calculate the evolution of the internal stellar temperature  $\tilde{T}(t)$ , the relative errors will be pretty small, about 6 times smaller than the indicated errors of the  $q$ -fits. The error of calculating the surface temperature  $T_s^\infty$  would be additionally twice smaller because  $T_s^\infty$  approximately behaves as  $T_s^\infty \propto \tilde{T}^{1/2}$  (Gudmundsson, Pethick & Epstein 1983). This is a well known property of the cooling theory: one can accurately calculate the neutron star temperature with not very accurate cooling functions. The temperature evolution appears to be really universal, almost independent of the EOS. The prize for that is also well known: even slight variations of the temperature correspond to sufficiently large variations of the neutrino cooling rates (e.g., Weisskopf et al. 2011). In other words, it is difficult to accurately determine the cooling rate from the data on the temperature.

In Fig. 1 the dependence (3) corresponds to slightly bent straight segments of the cooling curves. The neutrino cooling rate (6) is responsible for the MU and MUac curves, whereas the rate (8) for the SF and SFac curves. Equations (6) and (8) approximately describe the evolution of the internal temperature  $\tilde{T}$  in accordance with (3) (the lower and upper dotted  $\tilde{T}$ -lines, respectively). In or-





**Figure 3.** Solutions of the XMMU J1732 cooling problem in the  $M$ – $R$  plane. The lines correspond to fixed values of the suppression factor  $f_{tp}=0, 1/100, 1/80, 1/60$ , and  $1/40$  of the neutrino cooling rate by proton superfluidity for the models of heat blanketing envelopes containing carbon up to the density  $\rho_C = 10^{10}$  (left),  $3 \times 10^9$  (middle) and  $10^9$  g cm $^{-3}$  (right). Light shading shows the regions where  $f_{tp} \geq 1/60$ . The light dashed contours are the same as in Fig. 2. See text for details.

der to find the observed surface temperature  $T_s^\infty$  one should use the relation between the surface and internal temperatures of the star (Potekhin, Chabrier & Yakovlev 1997; Yakovlev et al. 2011).

### 3 COOLING OF XMMU 1732

In Paper I the observations of XMMU J1732 were interpreted using the carbon atmosphere models of neutron stars. For a wide range of possible masses  $M$  and radii  $R$  of neutron stars the observed spectra of XMMU J1732 were fitted with these theoretical models and the effective surface temperature  $T_s^\infty$  was determined. For  $d = 3.2$  kpc, the confidence contours of  $M$  and  $R$  at 50, 68 and 90 per cent significance levels are plotted in Fig. 2. The cross shows the best fit which corresponds to  $M = 1.53 M_\odot$ ,  $R = 12.4$  km and  $T_s^\infty = 1.78$  MK. The confidence contours restrict allowable masses and radii of XMMU J1732 for the assumed distance.

Let us show that the cooling theory further restricts allowable values of  $M$  and  $R$ . To this aim, for every pair of  $M$  and  $R$  we have constructed theoretical cooling curves using equations (3) and (5) as well as the relation between the internal and surface temperatures calculated by A. Potekhin (Yakovlev et al. 2011). The heat blanketing envelope is assumed to extend to the density  $\rho_b = 10^{10}$  g cm $^{-3}$ , and has the outside carbon layer (to a density  $\rho_C$ ) and an underlying layer of iron. In our case, the cooling is regulated by four parameters  $M$ ,  $R$ ,  $f_{tp}$  and  $\rho_C$ . Here we use  $\rho_C$ , which is more convenient than  $\Delta M_C$  used in Paper I. The relation between  $\Delta M_C$  and  $\rho_C$  is discussed in Appendix A.

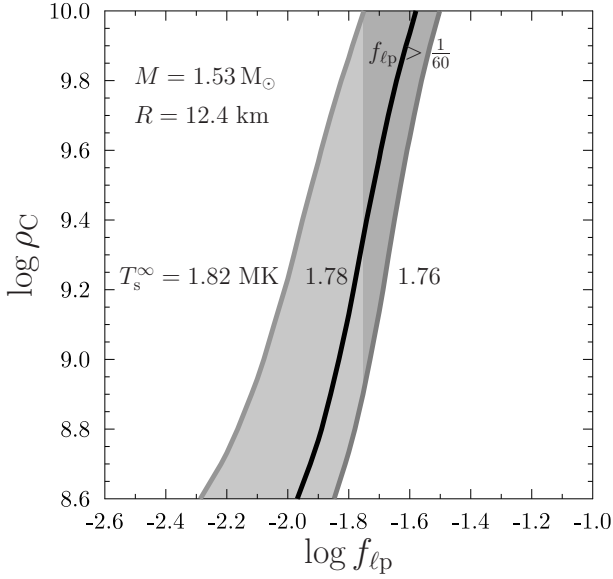
Now we can employ equation (3), combine it with the relation between the surface and internal temperatures, and calculate  $T_s^\infty$  for the assumed age of  $t = 27$  kyr. The presented formalism makes these calculations very simple, fast and straightforward (no need to run a sophisticated cooling code). For any pair of  $f_{tp}$  and  $\rho_C$ , we can now immediately find families of values of  $M$  and  $R$  which give the surface temperature  $T_s^\infty$  inferred in Paper I from the interpretation of the spectra. These values of  $M$  and  $R$  give us possible solutions of the reciprocal cooling problem for XMMU J1732. Such solutions are independent of the EOS in the stellar core because they are obtained with the universal approximations (6) and (8).

Fig. 3 shows several curves on the  $M$ – $R$  diagram which visualize the solutions of the reciprocal cooling problem. The left-hand

panel is for the most massive, fully carbon heat blanketing envelope with  $\rho_C = \rho_b = 10^{10}$  g cm $^{-3}$ . Each curve corresponds to a fixed value  $f_{tp}=0, 1/100, 1/80, 1/60$  and  $1/40$ . For weaker proton superfluidity (higher  $f_{tp}$ ) these curves shift to higher  $M$  and  $R$ . The middle panel shows the cooling solutions for the same values of  $f_{tp}$ , but at the smaller amount of carbon in the heat blanket,  $\rho_C = 3 \times 10^9$  g cm $^{-3}$ . At any fixed  $f_{tp}$  the decrease of  $\rho_C$  also shifts the curves to higher  $R$  and  $M$ . The right-hand panel is again for the same  $f_{tp}$  but for still smaller amount of carbon,  $\rho_C = 10^9$  g cm $^{-3}$ . The curves of constant  $f_{tp}$  continue shifting to higher  $M$  and  $R$ . Note that the curves in Fig. 3 are very sensitive to the approximations of  $q_{MU}$  and  $q_{SF}$ . This is a direct consequence of strong temperature dependence of the neutrino cooling function  $\ell(\bar{T})$  as detailed above.

The formal solutions of the cooling problem (Fig. 3) have to be reconciled with our knowledge of general properties of neutron star matter. The crucial parameter in our model is  $f_{tp}$ . It is determined by the profile of critical temperature  $T_{cp}(\rho)$  for the onset of proton superfluidity in the neutron star core. As mentioned above, the limiting value  $f_{tp} = 0$  corresponds to very strong proton superfluidity, with  $T_{cp}(\rho) \gtrsim 5 \times 10^9$  K everywhere in the core. Since theoretical values  $T_{cp}(\rho)$  are very model dependent (Lombardo & Schulze 2001) we would not like to rely on any specific model for proton superfluidity. Nevertheless, it is clear on theoretical grounds that the condition  $T_{cp}(\rho) \gtrsim 5 \times 10^9$  K in the entire core is unrealistic, especially in central regions of massive stars where the density of the matter is especially high. At very high densities, nuclear attraction of protons will inevitably turn into repulsion which should destroy proton superfluidity, increase the neutrino emission, and noticeably cool XMMU J1732, in disagreement with observations.

To summarize this discussion, very small values  $f_{tp} \rightarrow 0$  seem unrealistic, especially in massive stars. Realistic minimum values of  $f_{tp}$  have to be determined from a careful analysis of  $T_{cp}(\rho)$  calculations for the different models of nucleon interactions and various models of neutron stars which is beyond the scope of this paper. By way of illustration, we assume that  $f_{tp} \gtrsim 1/60$ . As seen from Fig. 3, this assumption would immediately impose serious constraints on mass and radius of XMMU J1732. For the fully carbon heat blanketing envelope (left-hand panel) we would have  $R > 10.5$  km and  $M > 1.2 M_\odot$ . For slightly less amount of carbon ( $\rho_C = 3 \times 10^9$  g cm $^{-3}$ , middle panel) we have  $R > 12$  km and  $M > 1.37 M_\odot$ . For



**Figure 4.** Solutions of the XMMU J1732 cooling problem in the  $f_{\ell p} - \rho_C$  plane for the star of  $M = 1.53 M_\odot$  and  $R = 12.4$  km with the measured surface temperature  $T_s^\infty = 1.78^{+0.04}_{-0.02}$  MK. The central thick black line corresponds to the best-fit surface temperature  $T_s^\infty = 1.78$  MK, while the other two lines are for  $T_s^\infty = 1.76$  and  $T_s^\infty = 1.82$  MK. Weakly shaded is the formal allowable range of  $f_{\ell p}$  and  $\rho_C$ . The densely shaded is a more realistic allowable range restricted by  $f_{\ell p} \geq 1/60$ . See text for details.

smaller amount of carbon ( $\rho_C = 10^9$  g cm $^{-3}$ , right-hand panel) the allowable values of  $M$  and  $R$  become uncomfortably high, barely compatible with the values derived from the spectral fits (Fig. 2). Therefore, it seems that the heat blankets with  $\rho_C \lesssim 10^9$  g cm $^{-3}$  are not favored by the cooling models of XMMU J1732, in agreement with the qualitative consideration of Paper I.

Let us add that according to numerous calculations and theoretical analysis of observations (e.g., Klähn et al. 2006; Beznogov & Yakovlev 2015 and references therein), the powerful direct Urca process of neutrino cooling really opens in sufficiently massive neutron stars. If it is not suppressed by strong superfluidity, it will not allow these stars to be as hot as XMMU J1732. From this point of view one can expect that XMMU J1732 is not very massive (say,  $M \lesssim 1.6 M_\odot$ ). Accordingly, one can further reduce the  $M$ – $R$  range allowed by cooling models (shaded regions in Fig. 3) by removing the region of rather high masses. However, this additional restriction of the cooling models requires further consideration. Generally, cooling theories predict that massive stars cannot be very hot. In principle, the direct Urca process can be suppressed by strong superfluidity but the existence of strong superfluidity in central parts of massive stars is unlikely on theoretical grounds (see above). However, XMMU J1732 can be a moderate-mass star, with the direct Urca process formally allowed but exponentially suppressed by strong superfluidity.

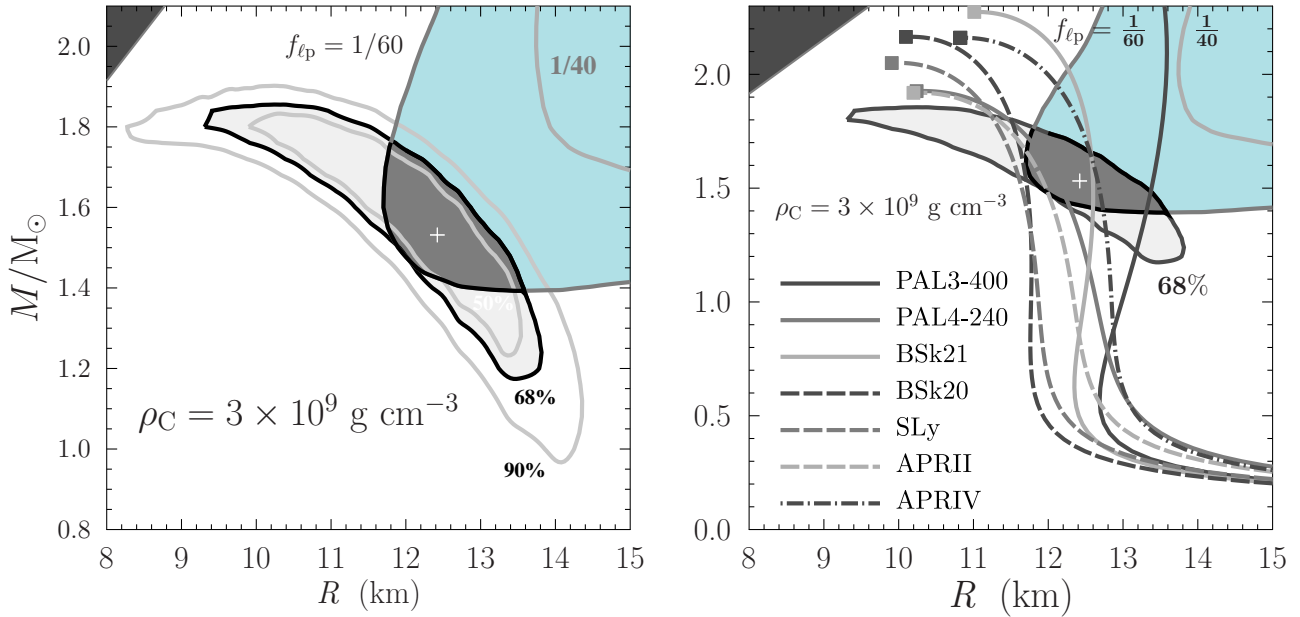
Let us recall that Paper I used the alternative description for the superfluid suppression of the neutrino cooling rate (by the parameter  $f_\ell$  instead of the parameter  $f_{\ell p}$  used here; see Section 2). The red dashed curve in fig. 8 of Paper I corresponds to  $f_\ell = 1/40$ . We have checked that it is qualitatively similar but not exactly coincident with the curve  $f_\ell = 1/40$  calculated using our current formalism at  $\rho_C = 3 \times 10^9$  g cm $^{-3}$ . The nature of the difference is twofold. Firstly, the dashed curve in Paper I is calculated for a con-

stant mass fraction of the carbon envelope,  $\Delta M_C/M$ , whereas here we take constant  $\rho_C$ . Secondly, here we employ the refined fit (6) for  $q_{\text{MU}}$ , whereas Paper I used slightly less accurate fit taken from Yakovlev et al. (2011).

Finally, Fig. 4 illustrates the sensitivity of extracting the neutrino cooling rate (the parameter  $f_{\ell p}$ ) from a measured surface temperature  $T_s^\infty$  of the star. To this aim, we have selected the best-fit neutron star model ( $M = 1.53 M_\odot$ ,  $R = 12.4$  km; crosses in Figs. 2 and 3) and assumed that the measurements give  $T_s^\infty = 1.78^{+0.04}_{-0.02}$  MK. The solid black central line gives formal solutions of the cooling problem in the  $f_{\ell p} - \rho_C$  plane. The line means the family of such solutions with different  $f_{\ell p}$  and  $\rho_C$  which give one and the same surface temperature  $T_s^\infty = 1.78$  MK. All of them are formally allowed by our cooling models. Two thick gray lines are similar solutions for the limiting temperatures  $T_s^\infty = 1.76$  and  $1.82$  MK. Weakly shaded region between the gray lines is the range of  $f_{\ell p}$  and  $\rho_C$  which is formally allowed at  $T_s^\infty = 1.78^{+0.04}_{-0.02}$  MK. We see that a narrow interval of  $T_s^\infty$  corresponds to rather wide intervals of  $f_{\ell p}$  and  $\rho_C$ . If we assume, on physical grounds, that  $f_{\ell p} \geq 1/60$  (as in Fig. 3), the allowable  $f_{\ell p} - \rho_C$  range would be much more restricted (densely shaded region in Fig. 4).

Now we can combine the mass-radius constraints given by spectral fits (Fig. 2) and the cooling theory (Fig. 3). One example of such analysis is presented in Fig. 5. To be specific, we take the 68 per cent banana-like confidence region of  $M$  and  $R$  from Fig. 2. For certainty, we assume the presence of carbon in the heat blanketing envelope up to the density  $\rho_C = 3 \times 10^9$  g cm $^{-3}$  (the shaded region of  $M$  and  $R$  in the middle panel of Fig. 3). The left-hand panel of Fig. 5 is plotted for the same ranges of  $M$  and  $R$  as in Fig. 3. It shows also the 50 and 90 per cent confidence contours from Fig. 2. The right-hand panel is plotted for a larger range of masses and presents also the mass-radius relations for the different EOSs from Fig. 2. As seen from Fig. 5, the cooling theory strongly reduces the banana-like  $M - R$  region. Let us mention that if we used only realistic theoretical EOSs of neutron stars (e.g., Haensel, Potekhin & Yakovlev 2007) to reduce the banana-like region, we would do similar reduction. The joint region allowed by the observed spectra and cooling models is densely shaded in Fig. 5. The cooling models disfavor very low and very large radii (both ends on the banana), just in accordance with theoretical expectations. Notice that the best fit  $M$  and  $R$  obtained from the spectral fits (the cross in Figs. 2, 3 and 5) appear to be near the center of the densely shaded region.

Our analysis based on Fig. 5 is evidently not strict because we have assumed specific values of  $\rho_C = 3 \times 10^9$  g cm $^{-3}$  and specific minimal  $f_{\ell p} = 1/60$ . We could slightly increase  $\rho_C$  (up to its maximum value  $\rho_C = 10^{10}$  g cm $^{-3}$ ) which would widen the range of  $M$  and  $R$  provided by the cooling models at  $f_{\ell p} \geq 1/60$  (left-hand panel on Fig. 3). However, this limiting case of the heat blanketing envelope fully composed of carbon seems not very realistic because at high densities carbon will start burning in pycnonuclear reactions (especially in carbon matter, containing admixture of other elements; see, e.g., Yakovlev et al. 2006). On the other hand, if we slightly decrease  $\rho_C$  below  $3 \times 10^9$  g cm $^{-3}$ , the  $M - R$  range allowed by the cooling models with  $f_{\ell p} \geq 1/60$  will shift to higher  $M$  and  $R$  (outside the banana range, where, in addition, the existence of exceptionally hot XMMU J1732 is questionable due to possible opening of the direct Urca process). In principle, we could take the minimum value of  $f_{\ell p}$  lower than  $1/60$ . This would increase the  $M - R$  range allowed by the cooling models but very low  $f_{\ell p}$  seem unrealistic (as explained above). All in all, although we have presented only one example of chosen  $\rho_C$  and  $f_{\ell p}$ , we have actually



**Figure 5.** Constraining  $M$  and  $R$  of XMMU J1732 from fitting the observed spectra (Fig. 2) and the cooling theory (Fig. 3). We employ the 68 per cent confidence region given by the spectral fits. As far as the cooling theory is concerned, we assume the carbon envelope with  $\rho_C = 3 \times 10^9 \text{ g cm}^{-3}$  and  $f_{lp} > 1/60$ . Densely shaded is the resulting confidence  $M - R$  region. Left-hand panel: same  $M - R$  scales as in Fig. 3, with the 50 and 90 per cent contours obtained from the spectral fits also shown. Right-hand panel: larger  $M$  range; the mass-radius relations for neutron stars with the different EOSs (Fig. 2) are added. See text for details.

not much freedom to vary the parameters and be consistent with the observations.

#### 4 CONCLUSIONS

We have extended the cooling theory of neutron stars to study a very slow cooling of exceptionally hot middle-aged stars. Such a cooling can take place (e.g., Paper I) under the effects of strong proton superfluidity in stellar cores (to suppress internal neutrino cooling) and large amount of sufficiently light elements in the heat blanketing envelopes (to increase the heat transparency of the envelopes and rise the surface temperature).

We have presented simple expressions (3), (5), (6) and (8) which enable fast and accurate calculation of the internal temperature  $\bar{T}(t)$  of a very slowly cooling neutron star as a function of age  $t$  at the neutrino cooling stage. The expressions are obtained for neutron stars with nucleon cores, where protons can be in a superfluid state. The expressions are universal, valid for many EOSs of nucleon matter. Equations (6) and (8) are derived by fitting the results of numerical calculations for many EOSs. Equation (8) takes into account a very slow (but finite) neutrino cooling in the presence of strong proton superfluidity which has not been calculated earlier. The effect of proton superfluidity is described by one parameter  $f_{lp}$  which is the suppression factor of the neutrino cooling of the star with respect to a standard neutrino candle. In our case the cooling is regulated by neutron star mass  $M$ , radius  $R$ , factor  $f_{lp}$ , and by the amount of light elements in the heat blanketing envelope.

The advantage of our approach is that it is universal and does not depend on a specific model for proton superfluidity. All the information on proton superfluidity is contained in  $f_{lp}$ . It is  $f_{lp}$  which can be inferred from observations; allowable models of  $T_{cp}$  to ensure this  $f_{lp}$  can be analyzed later. These models can describe the

cases in which a non-superfluid star would cool mainly via modified or even direct Urca process but both Urca processes are greatly suppressed by proton superfluidity. In this sense our consideration extends model-independent analysis of cooling neutron stars with standard cooling function  $\ell(T) \propto T^7$ , started by Yakovlev et al. (2011) and Weisskopf et al. (2011), and a more complicated model-independent analysis of the cooling enhanced by the onset of triplet-state pairing of neutrons and associated neutrino emission in the neutron star core (Shternin & Yakovlev 2015).

The cooling model has been applied to interpret the observations of thermal radiation of the XMMU J1732 neutron star in the supernova remnant HESS J1731–347. A preliminary interpretation was presented in Paper I. Following Paper I we have assumed the carbon atmosphere models, the distance  $d = 3.2 \text{ kpc}$ , and the neutron star age  $t = 27 \text{ kyr}$ . The modified theory has noticeably improved the interpretation of observations. We have obtained that the reasonable values of  $f_{lp}$  should be around  $1/60$  and the heat blanketing envelope should contain a lot of carbon, up to the density  $\rho_C \gtrsim 3 \times 10^9 \text{ g cm}^{-3}$ . The theory has allowed us to strongly restrict the range of masses and radii of XMMU J1732 (see the densely shaded region in Fig. 5) in comparison with the ranges obtained from spectral fits.

Nevertheless we would like to warn the reader that these results can be considered as semi-quantitative. For instance, strictly speaking, the factor  $f_{lp}$  can vary with time (larger layers of the core become superfluid). We have neglected this effect assuming it is weak. Moreover, owing to the strong temperature dependence of the neutrino cooling function  $\ell(T)$  (Section 2), the values of  $f_{lp}$  which we infer from the observations are very sensitive to the measured values of  $T_s^\infty$  and to a not very certain microphysics of the neutron star matter. In particular, they are sensitive to the thermal insulation of the heat blanketing envelopes (to the relation between the surface and internal temperatures). Another example –

our equations (6) and (8) are obtained using certain expressions (e.g., Yakovlev et al. 2001) for the neutrino emission in the modified Urca process and nucleon-nucleon collisions (Table 1). Although these expressions are widely used in cooling simulations, they are model dependent. Were the theory of neutrino processes improved (first of all, with regard to matrix elements of the processes), equations (6) and (8) should have been updated which may change the results.

In addition, our consideration is based on the age of XMMU J1732 equal to 27 kyr. However, one cannot exclude that the age is different which would affect the results. If the age were larger, say, 40 kyr, our cooling model would still be able to explain the data but assuming the strongest proton superfluidity and fully carbon blanketing envelope. Were the age lower (e.g., as low as 10 kyr) the situation would be more relaxed, than at  $t = 27$  kyr, but we would still need both, strong superfluidity and massive carbon envelope. To become an ‘ordinary’ cooling neutron star (instead of extraordinary hot one) its age should be  $t \lesssim 3$  kyr. The distance to XMMU J1732 is also not very certain. Were  $d = 4.5$  kpc instead of 3.2 kpc, the values of  $M$  and  $R$  inferred from the spectral fits would be noticeable higher, not very realistic for neutron stars, and the inferred surface temperature would also be slightly higher (Paper I). Although there is no rigorous proof, it is widely believed that such a very massive star should cool rapidly, in disagreement with the inferred  $T_s^\infty$ .

## ACKNOWLEDGEMENTS

The work of AK and DY was partly supported by Russian Foundation for Basic Research (grants Nos. 14-02-00868-a and 13-02-12017-ofi-M) and the work of VS by DFG grant WE 1312/48-1.

## REFERENCES

- Antoniadis J. et al., 2013, *Science*, 340, 448  
 Beznogov M. V., Yakovlev D. G., 2015, *MNRAS*, 447, 1598  
 Demorest P. B., Pennucci T., Ransom S. M., Roberts M. S. E., Hessels J. W. T., 2010, *Nature*, 467, 1081  
 Flowers E., Ruderman M., Sutherland P., 1976, *ApJ*, 205, 541  
 Gudmundsson E. H., Pethick C. J., Epstein R. I., 1983, *ApJ*, 272, 286  
 Gusakov M. E., Kaminker A. D., Yakovlev D. G., Gnedin O. Y., 2005, *MNRAS*, 363, 555  
 Haensel P., Potekhin A. Y., Yakovlev D. G., 2007, *Neutron Stars. 1. Equation of State and Structure*. Springer, New York  
 Halpern J. P., Gotthelf E. V., 2010, *ApJ*, 710, 941  
 Ho W. C. G., Heinke C. O., 2009, *Nature*, 462, 71  
 Kaminker A. D., Kaurov A. A., Potekhin A. Y., Yakovlev D. G., 2014, *MNRAS*, 442, 3484  
 Kaminker A. D., Pethick C. J., Potekhin A. Y., Thorsson V., Yakovlev D. G., 1999, *A&A*, 343, 1009  
 Klähn T. et al., 2006, *Phys. Rev. C*, 74, 035802  
 Klochkov D., Pühlhofer G., Suleimanov V., Simon S., Werner K., Santangelo A., 2013, *A&A*, 556, A41  
 Klochkov D., Suleimanov V., Pühlhofer G., Yakovlev D. G., Santangelo A., Werner K., 2015, *A&A*, 573, A53 (Paper I)  
 Lattimer J. M., Pethick C. J., Prakash M., Haensel P., 1991, *Phys. Rev. Lett.*, 66, 2701  
 Leinson L. B., Pérez A., 2006, *Physics Letters B*, 638, 114

- Lombardo U., Schulze H.-J., 2001, in *Lecture Notes in Physics*, Berlin Springer Verlag, Vol. 578, Physics of Neutron Star Interiors, Blaschke D., Glendenning N. K., Sedrakian A., eds., p. 30  
 Ofengeim D. D., Kaminker A. D., Yakovlev D. G., 2014, *Europhysics Lett.*, 108, 31002  
 Page D., Lattimer J. M., Prakash M., Steiner A. W., 2009, *ApJ*, 707, 1131  
 Potekhin A. Y., Chabrier G., Yakovlev D. G., 1997, *A&A*, 323, 415  
 Potekhin A. Y., Fantina A. F., Chamel N., Pearson J. M., Goriely S., 2013, *A&A*, 560, A48  
 Shternin P. S., Yakovlev D. G., 2015, *MNRAS*, 446, 3621  
 Suleimanov V. F., Klochkov D., Pavlov G. G., Werner K., 2014, *ApJ Suppl.*, 210, 13  
 Tian W. W., Leahy D. A., Haverkorn M., Jiang B., 2008, *ApJ*, 679, L85  
 Tian W. W., Li Z., Leahy D. A., Yang J., Yang X. J., Yamazaki R., Lu D., 2010, *ApJ*, 712, 790  
 Weisskopf M. C., Tennant A. F., Yakovlev D. G., Harding A., Zavlin V. E., O’Dell S. L., Elsner R. F., Becker W., 2011, *ApJ*, 743, 139  
 Yakovlev D. G., Gasques L. R., Afanasjev A. V., Beard M., Wiescher M., 2006, *Phys. Rev. C*, 74, 035803  
 Yakovlev D. G., Ho W. C. G., Shternin P. S., Heinke C. O., Potekhin A. Y., 2011, *MNRAS*, 411, 1977  
 Yakovlev D. G., Kaminker A. D., Gnedin O. Y., Haensel P., 2001, *Phys. Rep.*, 354, 1  
 Yakovlev D. G., Pethick C. J., 2004, *Annu. Rev. Astron. Astrophys.*, 42, 169

## APPENDIX A: PARAMETERS OF CARBON ENVELOPE

A carbon ( $^{12}\text{C}$ ) layer in the heat blanketing envelope of a neutron star can be characterized by the density  $\rho_C$  at the bottom of this layer. Alternatively, the layer can be specified by the ratio  $\Delta M_C/M$  of the gravitational masses of the layer,  $\Delta M_C$ , and of the entire star,  $M$ . These quantities are known to be related as (e.g., Potekhin, Chabrier & Yakovlev 1997)

$$\frac{\Delta M_C}{M} = \frac{P(\rho_C)}{P_0 g_{14}^2} = \frac{1.510 \times 10^{-11}}{g_{14}^2} \times \left[ x_r \sqrt{1 + x_r^2} \left( \frac{2}{3} x_r^2 - 1 \right) + \ln \left( x_r + \sqrt{1 + x_r^2} \right) \right], \quad (\text{A1})$$

where  $g_{14}$  is the surface gravity in units of  $10^{14} \text{ cm s}^{-2}$ ,  $P_0 = 1.193 \times 10^{34} \text{ dyn cm}^{-2}$ ,  $x_r = 1.009 (\rho_6/\mu_e)^{1/3}$  is the relativistic parameter of degenerate electrons,  $\mu_e = A/Z = 12/6 = 2$  is the number of nucleons per one electron in carbon matter,  $\rho_6 = \rho_C/(10^6 \text{ g cm}^{-3})$ , and  $P(\rho_C)$  is the pressure at the bottom of the carbon layer which is approximated by the pressure of free degenerate relativistic electrons; e.g., Haensel, Potekhin & Yakovlev (2007). For a star with  $M = 1.4 M_\odot$  and  $R = 12 \text{ km}$  ( $g_{14} = 1.59$ ,  $x = r_g/R = 0.345$ ) at  $\rho_C = 10^6, 10^8$  and  $10^{10} \text{ g cm}^{-3}$  one has  $\Delta M_C/M = 8.680 \times 10^{-13}, 7.105 \times 10^{-10}$  and  $3.500 \times 10^{-7}$ , respectively.

In addition, the layer can be characterized by the column baryon mass density of carbon,  $\Sigma$ , which is related to  $\Delta M_C$  as

$$\Sigma = \frac{\Delta M_C}{4\pi R^2 \sqrt{1 - x}}. \quad (\text{A2})$$

The dependence of  $\Sigma$  and  $\Delta M_C/M$  on  $\rho_C$  is, of course, similar.



## APPENDIX B: ANALYTIC APPROXIMATION OF NEUTRINO LUMINOSITY OF CRUST

Let us obtain an analytic approximation for the neutrino luminosity of a neutron star crust in the reference frame of distant observer,

$$L_{\text{cr}} = \int_{\text{crust}} Q \exp(2\Phi) dV, \quad (\text{B1})$$

where  $Q$  is a local neutrino emissivity,  $\exp(2\Phi) = g_{00}(r)$  is a time-like component of the metric tensor,  $r$  is a circumferential radius, and  $dV$  a proper volume element. The integration is carried out over the crust volume.

Assume that the neutron star is thermally relaxed. Then the crust is nearly isothermal;  $\tilde{T}$  is constant over the entire crust excluding a thin outer heat blanketing envelope whose contribution to  $L_{\text{cr}}$  is negligible. In this case the local temperature is given by  $T(r) = \tilde{T} \exp(-\Phi)$ . Because the crust is thin (with the thickness of  $\sim 0.1R$ ) and contains  $\sim 0.01$  of the neutron star mass, its structure can be calculated in the relativistic Cowling approximation; it is nearly independent of the EOS in the stellar core. The metric function in the crust can be approximated as  $g_{00} = 1 - r_g/r$ , and the pressure gradient as

$$\frac{dP}{dr} = -\frac{G\rho M}{r^2} \frac{1}{1 - r_g/r}. \quad (\text{B2})$$

We replace the integration over  $r$  in (B1) by the integration over a new variable  $s$  defined as

$$s(P) = \int_0^P \frac{dP}{\rho c^2}, \quad (\text{B3})$$

where  $P = P(r)$ . Then one can obtain

$$L_{\text{cr}} = 8\pi r_g^3 \gamma^5 \int_{s_b}^{s_{\text{cc}}} \frac{Q \exp[-3(s - s_b)]}{\{\gamma^2 - \exp[-2(s - s_b)]\}^4} ds; \quad (\text{B4})$$

$s_b$  corresponds to the bottom of the heat blanketing envelope ( $\rho = \rho_b = 10^{10} \text{ g cm}^{-3}$ ), and  $s_{\text{cc}}$  to the crust-core interface ( $\rho_{\text{cc}} \approx 1.5 \times 10^{14} \text{ g cm}^{-3}$ );  $\gamma$  is given by equation (7).

Let us consider the most important case in which the main contribution to  $L_{\text{cr}}$  comes from the neutrino-pair bremsstrahlung due to collisions of strongly degenerate relativistic electrons with atomic nuclei. The neutrino emissivity of this process has been thoroughly investigated. Schematically, it can be written as  $Q \propto T^6 \Lambda(\rho, T)$ , where  $\Lambda$  is a Coulomb logarithm (a weakly varying function of  $T$  and  $\rho$ ), while the normalization factor depends only on  $\rho$ . In addition, the Coulomb logarithm behaves smoothly at sufficiently high densities which give the main contribution to  $L_{\text{cr}}$  (see, e.g., Kaminker et al. 1999; Ofengeim, Kaminker & Yakovlev 2014). With this in mind one can show that at temperatures  $\tilde{T} \sim 3 \times 10^7 - 10^9 \text{ K}$  of practical interest the neutrino luminosity  $L_{\text{cr}}$  can be accurately approximated as

$$L_{\text{cr}} = 9.053 \times 10^{34} \text{ erg s}^{-1} \left( \frac{M}{M_{\odot}} \right)^3 \frac{\tilde{T}_9^6 \gamma^{11}}{(\gamma^2 - 1)^4} \frac{\phi(y)}{\psi(z)}, \quad (\text{B5})$$

where  $y = \tilde{T}_9 \gamma / 1.0643$ ,  $z = \gamma^2 - 1$ ,

$$\phi(y) = ay \frac{(y-1) \exp(-py) + 1}{p+1}, \quad a = 1.7, \quad p = 13.0; \quad (\text{B6})$$

$$\psi(z) = \left( \frac{b}{z} - 1 \right) \exp(-qz) + 1, \quad b = 0.1, \quad q = 15.0. \quad (\text{B7})$$

The approximation has been obtained using the smooth composition model of the ground-state crust (Haensel, Potekhin & Yakovlev 2007).

We have compared the approximated  $L_{\text{cr}}$ , equation (B5), with that calculated numerically from equation (B4). The comparison has been made for the models of the neutron star crust with the FPS or SLy EOS (Haensel, Potekhin & Yakovlev 2007) but with the same description of the nuclear composition as in (B5). We have considered the wide ranges of masses  $1.0 M_{\odot} \leq M \leq 2.5 M_{\odot}$ , radii  $10 \leq R \leq 16 \text{ km}$  and internal temperatures  $3 \times 10^7 \leq \tilde{T} \leq 10^9 \text{ K}$ . The relative deviations of approximated and numerical values of  $\log L_{\text{cr}} [\text{erg s}^{-1}]$  do not exceed 2–3 per cent. In addition, we have calculated  $L_{\text{cr}}$  for the  $1.4 M_{\odot}$  neutron star model with the BSk21 EOS in the core and compared with  $L_{\text{cr}}$  given by equation (B5). The relative deviations turn out to be as small as above. Furthermore, we have checked that the neutrino emissivity due to the electron bremsstrahlung on atomic nuclei in the crust made of BSk20 or BSk21 EOS taken with its proper nuclear composition (Potekhin et al. 2013) is almost the same as for the smooth-composition model used in equation (B5).

The above analysis demonstrates that our approximation is almost universal. It is nearly independent of the EOS of superdense matter in the neutron star core. Moreover, according to our calculations, the main contribution to  $L_{\text{cr}}$  comes from the density range  $10^{13} \text{ g cm}^{-3} \lesssim \rho \lesssim \rho_{\text{cc}}$ . In this range the models of accreted crust are almost indistinguishable (Haensel, Potekhin & Yakovlev 2007; Ofengeim, Kaminker & Yakovlev 2014) from the models of ground-state crust (cold-catalyzed matter). Therefore,  $L_{\text{cr}}$  should be weakly dependent of the EOS in the crust.

As XMMU J1732 is thought to undergo extremely slow neutrino cooling, one might think that the neutrino emission from its crust can be important along with the neutrino emission due to nn-collisions in the core. However, simple estimates based on equation (B5) show that for the range of  $M$  and  $R$  of our interest (Section 3)  $L_{\text{cr}}$  is typically by more than 10 times lower than the neutrino luminosity due to nn-collisions in the core. Therefore, it can be disregarded for the XMMU J1732 cooling problem.

Nevertheless, one might speculate on the existence of very low-mass neutron stars, with masses of  $\sim (0.1 - 0.6) M_{\odot}$ . These stars are hypothetical, difficult to produce on evolutionary grounds. If, however, they exist, they would have large radii and bulky crusts (Haensel, Potekhin & Yakovlev 2007). Their cooling can be regulated by neutrino emission from the crust given by equation (B5).